

# Construction of $(h,k)$ -Coterie Quorum System Based on Majority Coterie

*by* Lawi Armin

---

**Submission date:** 19-Feb-2020 11:51AM (UTC+0700)

**Submission ID:** 1259987015

**File name:** Arsal\_2019\_J.\_Phys.\_\_Conf.\_Ser.\_1341\_042019.pdf (1M)

**Word count:** 2318

**Character count:** 12579

**PAPER · OPEN ACCESS**

## Construction of (h,k)-Coterie Quorum System Based on Majority Coterie

3

To cite this article: A Aarsal *et al* 2019 *J. Phys.: Conf. Ser.* **1341** 042019View the [article online](#) for updates and enhancements.**IOP | ebooks™**Bringing you innovative digital publishing with leading voices  
to create your essential collection of books in STEM research.Start exploring the collection - download the first chapter of  
every title for free.

## Construction of $(h,k)$ -Coterie Quorum System Based on Majority Coterie

A Arsal<sup>1,a</sup>, A Lawi<sup>1,b</sup> and A K Amir<sup>3,c</sup>

<sup>1</sup>Mathematics Department, Hasanuddin University, Indonesia

<sup>a</sup>armayaniarsal@gmail.com, <sup>b</sup>armin@unhas.ac.id, <sup>c</sup>amirkamalimir@yahoo.com

**Abstract.** Quorum system of  $(h,k)$ -majority coterie is a set system which elements are a collection of sets  $k$ -coterie provided that each element satisfies bicoterie and disjoint properties. Some of related studies have tried to make the construction of this quorum system but constrained by the problem of generalization. In this paper, to overcome the problem we first compile an equation to determine the size of quorum. Then we arrange quorums that satisfies the equation in a quorum system. The way are (a) divide the universe set into  $m$  parts so that  $h$  parts are separated, (b) construct a quorum that satisfy  $k$ -coterie, (c) construct a quorum system that satisfy bicoterie and disjoint properties.

### 1. Introduction

A quorum is a set that contains elements of the universal set, either in part or in whole. Quorums that satisfy the intersection and minimality properties forms a set called coterie. Coterie has an important role in resolving resource allocation problems, such as mutual exclusion issues. Therefore, many researchers have developed the concept of quorum and coterie.

First, [1] introduced a collection of coterie sets with intersection and minimality properties that bind its elements. Furthermore, various kinds of coterie appear, including majority coterie which add conditions, namely the number of elements of the quorum must satisfy the equation  $|Q| = \left\lfloor \frac{n}{2} \right\rfloor + 1$ . Furthermore, [2] conducts research related to the development of coterie and finds a collection of new sets called  $k$ -coterie.

Besides coterie, the development of the quorum system is also an interesting thing to study. In 1993, [3] introduced a set system consisting of a pair of coterie called bicoterie. Furthermore, the idea of developing the concept of bicoterie was born into a set system with more than two elements. In 2004, [4] introduced the  $(m,1,k)$ -coterie quorum system, but was limited to  $h = 1$ . Finally [5] succeeded in answering the limitations of  $h$  by the discovery of a new set systems called  $(h,k)$ -coterie. Furthermore, [6] uses quorum  $(h,k)$ -coterie approach and forms another system called  $(n,m,k,d)$ . But this concept can only be used in the universe set  $P$  whose number of elements is a squared number or  $|P| = a^2$ .

In this paper, a system of quorum  $(h,k)$ -coterie will be constructed based on majority coterie with a more general concept. The way of construction must begin with compiling equations to determine the size of quorum. Then by following the formulation phase of the equation there will be a quorum system construction process that satisfy the membership requirements for the quorum  $(m,h,k)$ -coterie system. Finally, a more general construction of the quorum system can be obtained.

### 2. Quorum System



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

In this section, we will discuss a kind of coterie that related with this research. Let  $P$  is a universe set with  $n$  elements.

**Definition 2.1 (Coterie [1]).** A nonempty set  $C (\subseteq 2^P)$  is coterie on  $P$  iff satisfies the following properties:

1. Intersection :  $\forall Q, Q' \in C$  berlaku  $Q \cap Q' \neq \emptyset$
2. Minimality :  $\forall Q, Q' \in C$  berlaku  $Q \not\subseteq Q'$

**Example 2.1.** Set  $C = \{\{1,2\}, \{2,3\}\}$  is coterie under  $P = \{1,2,3\}$

**Definition 2.2 (Majority Coterie).** A set collection  $M$  called majority coterie under  $P$ , with  $n = |P|$  iff satisfies

$$M = \left\{ Q \mid |Q| = \left\lfloor \frac{n}{2} \right\rfloor + 1, \quad Q \subseteq P \right\}$$

**Example 2.2.** Set collection  $M = \{\{1,2\}, \{1,3\}, \{2,3\}\}$  is a majority coterie under  $P = \{1,2,3\}$ .

**Definition 2.3 ( $k$ -Coterie [2]).** Let  $k > 1$ . A nonempty  $C$  under set  $P$  called  $k$ -koteri under  $P$  iff  $C$  satisfies the following properties:

1. Non-intersection:  $\forall Q_1, Q_2, \dots, Q_d \in C$  which non-intersection with  $d < k$ , namely  $Q_i \cap Q_j = \emptyset, 1 \leq i \neq j \leq d, \exists Q \in C \ni Q \cap Q_i = \emptyset, 1 \leq i \leq d$ .
2. Intersection:  $\forall Q_1, Q_2, \dots, Q_p \in C, p > k, \exists (Q_i, Q_j) \ni Q_i \cap Q_j \neq \emptyset, 1 \leq i \neq j \leq p$ .
3. Minimality:  $\forall Q_i, Q_j \in C, Q_i \not\subseteq Q_j$ .

Quorum system 1-coterie called coterie.

Number of elements for  $k$ -coterie satisfies equation

$$|Q| = \left\lfloor \frac{n}{k+1} \right\rfloor + 1 \quad (1)$$

**Example 2.3.** Set system  $C = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}\}$  is 2-coterie under  $P = \{1,2,3,4\}$ .

**Definition 2.4 (Bicoterie [3]).** Let  $C_1$  and  $C_2$  are sets of subsets of  $P$ . Set system  $B = \{C_1, C_2\}$  called bicoterie under  $P$  iff satisfy:

1. Intersection:  $\forall Q \in C_1, \forall Q' \in C_2$  then  $Q \cap Q' \neq \emptyset$ .
2. Minimality :  $\forall Q \in C_1, \forall Q' \in C_2$  then  $Q \not\subseteq Q'$ .

**Example 2.4.** Set system  $B = \{C_1, C_2\}$ , with  $C_1 = \{\{1,2\}, \{3,4\}\}$  and  $C_2 = \{\{1,3\}, \{2,4\}\}$  is bicoterie under  $P = \{1,2,3,4\}$ .

**Definition 2.5 ( $(h,k)$ -Coterie [5]).** A collection of sets  $B = \{C_1, C_2, \dots, C_m\}$ , where  $C_i$  is a  $k$ -coterie under  $P, \forall C_i \in B$  is an  $(h, k)$ -coterie under  $P$  iff the following conditions hold:

1. Disjoint: for any  $l (< h)$  mutually disjoint elements  $C_1, C_2, \dots, C_l \in B$ , there is another element  $C \in B$  such that  $C$  dan  $C_i$  are disjoint for all  $i = 1, 2, \dots, l$ .
2. Bicoterie:  $\forall C_1, C_2, \dots, C_h, C_{h+1} \in B$  there exists a pair  $\{C_i, C_j\}$  forms bicoterie,  $\forall 1 \leq i \neq j \leq h + 1$ .

This construction can only be used if number of  $(h, k)$  equals to  $\left(\frac{m}{2}, \sqrt{\frac{n}{m}}\right)$ , where  $n$  are number of elements of  $P$ .

**Example 2.5.** Quorum system  $B = \{C_1, C_2, C_3, C_4\}$  where:

$$C_1 = \{\{1,3,5,7\}, \{9,11,13,15\}\},$$

$$C_2 = \{\{1,2,9,10\}, \{3,4,11,12\}\},$$

$$C_3 = \{\{2,4,6,8\}, \{10,12,14,16\}\},$$

$$C_4 = \{\{5,6,13,14\}, \{7,8,15,16\}\},$$

is (2,2)-coterie where  $m = 4$  under  $P = \{1,2,3, \dots, 16\}$ .

### 3. $(n, m, h, k)$ -Resource Allocation Problem

A distributed system consisting of  $m$  resources will be allocated to  $n$  processes provided that the degree of concurrency ( $hk$ ) is reached. Degree of concurrency ( $hk$ ) means that at one time there are  $h$  active resources, from  $m$  available resources, which can be accessed simultaneously by each  $k$  process. The problem in the system  $(n, m, h, k)$ -allocated resources is the determination of the values of  $n$  and  $m$  which can guarantee that each active resource  $h$  can be allocated in an orderly manner and that every  $k$ -process that wants to access resources can served at one time.

The relationship between processe-access-resource in  $(n, m, h, k)$ -allocated resources problems can be expressed in the Complete Resource Allocation Graph (CRAG). In this case CRAG involves the set of processes  $P$  and the set of resources  $R$  forming a bipartition graph  $G = (V, E)$ , with  $V = P \cup R$  and  $E$  is the set of sides. An element  $e = (p, r) \in E$  is said to be the side if and only if the process  $p$  requests access to the resource  $r$ .

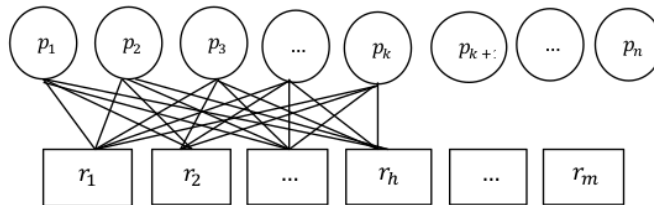


Figure 1. CRAG of  $(n, m, h, k)$ -allocated resources problems

Configurations of edge in CRAG can be described in various forms as long as the degree of concurrency is satisfie.

To solve the  $(n, m, h, k)$ -allocated resources problem, it is necessary to determine the structure and size of the system involving the set  $n$  and  $m$ . Solution to this problem is to form a quorum system  $(h, k)$ -majority district.

### 4. Construction of Quorum System $(h, k)$ - Majority Coterie

The quorum system  $(h, k)$ -majority coterie is a set system whose elements are collections of sets of  $k$ -coterie that satisfies the properties, namely there are  $h$  elements that are mutually disjoint and every  $h + 1$  elements which mutually bicoterie.

**Definition 4.1  $(h, k)$ -Majority Coterie.** Set system  $B = \{C_1, C_2, \dots, C_m\}$  which satisfies disjoint and bicoterie properties called  $(h, k)$ -majority coterie iff  $\forall C_i \in B, i = 1, 2, \dots, m$  satisfies:

$$C_i = \left\{ Q \mid |Q| = \left\lceil \frac{\left\lfloor \frac{n}{m} \right\rfloor + 1}{k + 1} \right\rceil + (m - h - 1)k \right\}$$

when  $m \geq 2h$  and  $n \geq hk|Q|$ .

#### 4.1. Size of Quorum

Size of quorum or number of elements each quorum in  $(m, h, k)$ -coterie satisfies

$$|Q| = \left\lfloor \frac{\left\lfloor \frac{n}{m} \right\rfloor + 1}{k+1} \right\rfloor + (m-h-1)k \quad (2)$$

This equation is obtained by following these steps:

1. Find the number of elements for each  $C_i$

Suppose that the universe set of  $P$  consists of  $n$  elements then every  $C_i, i = 1, 2, \dots, m$  consists of  $\left\lfloor \frac{n}{m} \right\rfloor$  elements.

$$C_1 = \{Q_1 \mid |Q_1| = \left\lfloor \frac{n}{m} \right\rfloor, Q_1 \subseteq P_1\}$$

$$C_2 = \{Q_2 \mid |Q_2| = \left\lfloor \frac{n}{m} \right\rfloor, Q_2 \subseteq P_2\}$$

⋮

$$C_m = \{Q_m \mid |Q_m| = \left\lfloor \frac{n}{m} \right\rfloor, Q_m \subseteq P_m\}$$

2. Find the number of elements each quorum in  $C_m$

In order for the  $k$ -koteri properties to be satisfied, each quorum in  $C_i$  must satisfy the equation

$$|Q| = \left\lfloor \frac{\left\lfloor \frac{n}{m} \right\rfloor + 1}{k+1} \right\rfloor \quad (3)$$

Each quorum in step 1 is separated from each other, so it is necessary to combine the universe set to satisfy the properties of bicoterie. Based on definition 2.5, it is known that each coterie will pair to form a bicoterie with  $(m-h-1)$  other coterie in  $m$

$$\left[ \begin{array}{l} C_1 = \{Q_{11}, Q_{12}, \dots, Q_{1k}, \dots, Q_{1p} \mid Q_{1p} \subseteq P_1\} \\ C_2 = \{Q_{21}, Q_{22}, \dots, Q_{2k}, \dots, Q_{2p} \mid Q_{2p} \subseteq P_2\} \\ C_3 = \{Q_{31}, Q_{32}, \dots, Q_{3k}, \dots, Q_{3p} \mid Q_{3p} \subseteq P_3\} \\ \vdots \\ C_{(m-h)} = \{Q_{(m-h)1}, Q_{(m-h)2}, \dots, Q_{(m-h)k}, \\ \dots, Q_{(m-h)p} \mid Q_{(m-h)p} \subseteq P_{(m-h)}\} \\ \vdots \\ C_m = \{Q_{m1}, Q_{m2}, \dots, Q_{mk}, \dots, Q_{mp} \mid Q_{mp} \subseteq P_m\} \end{array} \right.$$

This will affect the number of elements of the universe set  $P$  of each coterie and the number of elements of coterie. So the  $k$ -koteri obtained at this step comes from the following universe set.

$$P_1^* = P_1 \cup P_2 \cup P_3 \cup \dots \cup P_{m-h}$$

$$P_2^* = P_2 \cup P_3 \cup P_4 \cup \dots \cup P_{m-h+1}$$

⋮

$$P_m^* = P_m \cup P_1 \cup P_2 \cup \dots \cup P_{m-h-1}$$

Finally we can find

$$C_1 = \{Q_{11}, Q_{12}, \dots, Q_{1k}, \dots, Q_{1p} \mid Q_{1p} \subseteq P_1^*\}$$

$$C_2 = \{Q_{21}, Q_{22}, \dots, Q_{2k}, \dots, Q_{2p} \mid Q_{2p} \subseteq P_2^*\}$$

⋮

$$C_m = \{Q_{m1}, Q_{m2}, \dots, Q_{mk}, \dots, Q_{mp} \mid Q_{mp} \subseteq P_m^*\}$$

3. Find the number of elements of quorum for quorum system  $(m, h, k)$ -coterie

In quorum system  $(m, h, k)$ -coterie, each collection sets  $C_1, C_2, \dots, C_m$  in step 2 must be satisfy disjoint properties. It is say that there are  $h$  ( $h < m$ ) element ( $C_1, C_2, \dots, C_h$ ) mutually disjoint.

Because of union of universe set in step 2 make the size of quorum to increase. The increasing number of elements can be investigated from the number of coterie which mutually bicoterie.

If  $Q_1 \subseteq P_1, Q_2 \subseteq P_2, Q_3 \subseteq P_3, \dots, Q_{m-h} \subseteq P_{m-h}$ , then each quorum in  $C_2, C_3, \dots, C_{(m-h)}$  will join into each quorum in  $C_1$  such as  $Q_1 \cap Q_2 \neq \emptyset, Q_1 \cap Q_3 \neq \emptyset, \dots, Q_1 \cap Q_{m-h} \neq \emptyset$ . Finally,  $\forall Q_i \in C_i, i = 1, 2, \dots, m$  we can get additional element of quorum at least  $k$  from each pair bicoterie. So the equation (3.3) becomes

$$|Q| = \left\lceil \frac{\left\lfloor \frac{n}{m} \right\rfloor + 1}{k + 1} \right\rceil + (m - h - 1)k \quad (4)$$

where  $\frac{n}{m} > k$  and  $m \geq 2h$ .

Other impacts from this join is exist quorums in coterie mutually subset. Therefore we need selection quorum step for satisfie disjoint properties. But this step not change the quorum size.

#### 4.2. Construction of Quorum System

The way of construction of quorum system are:

1. Find the element of universe set  $P_i$

Let universe set  $P$  contain  $n$  element then the number of element  $P_i = \left\lfloor \frac{n}{m} \right\rfloor$  where  $i = 1, 2, \dots, m$ .

2. Arrange  $P_i$  so that we can get coterie which satisfies disjoint and bicoterie property.

In order for the quorum system to satisfy the disjoint property, must exist a universe set  $P_i$  that is separated from several other universe sets, depending  $h$ . This means

$$\exists P_1 \cap P_2 \cap \dots \cap P_h = \emptyset$$

In other hand, to satisfy bicoterie property then must exist  $P_i$  which mutually bicoterie with several other sets, depending  $h$ . This means

$$\exists P_1, P_{h+1}, P_{h+2}, \dots, P_m \ni P_1 \cap P_{h+1} \neq \emptyset, P_1 \cap P_{h+2} \neq \emptyset, \dots, P_1 \cap P_m \neq \emptyset$$

Let  $C_i \subseteq P_i$  and  $C_j \subseteq P_j$  mutually bicoterie then element of set  $P_i$  and  $P_p$  arranged such as  $|P_i \cap P_p| = k^2$ . In this step we can find a new  $P_i^*$  that satisfy bicoterie and disjoint property where  $|P_i^*| = \left\lfloor \frac{n}{h} \right\rfloor$  and  $|P_i^*| \geq k|Q|$ . Because  $|P_i^*| = \left\lfloor \frac{n}{h} \right\rfloor$  and  $|P_i^*| \geq k|Q|$  then we can conclude

$$\begin{aligned} \left\lfloor \frac{n}{h} \right\rfloor &\geq k|Q| \\ n &\geq |Q| \cdot hk \end{aligned} \quad (5)$$

3. Construction coterie which satisfy  $k$ -coterie property and quorum size  $|Q|$

In this step, we use  $P_i^*$  to arrange coterie  $C_i^* \in 2^{P_i^*}, i = 1, 2, \dots, m$

$$\begin{aligned} C_1^* &= \{Q_1 \mid Q_1 = \{C_{|Q|}^{|P_1^*|}\}\} \\ C_2^* &= \{Q_2 \mid Q_2 = \{C_{|Q|}^{|P_2^*|}\}\} \\ &\vdots \\ C_m^* &= \{Q_m \mid Q_m = \{C_{|Q|}^{|P_m^*|}\}\} \end{aligned}$$

where  $C_{|Q|}^{|P_i^*|}$  is combination  $|Q|$  of  $P_i^*$ ,  $i = 1, 2, \dots, m$ .

4. Construction quorum system which satisfy bicoterie and disjoint property.

In this step, we will selection quorum that we get from previous step.

Let  $P_i \cap P_j = \emptyset$ , that means if  $C_i \in 2^{P_i}$  and  $C_j \in 2^{P_j}$  then  $C_i$  and  $C_j$  mutually disjoint for  $i \neq j, i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, m$ . So that, each arrange  $k$ -coterie will cause  $C_i$  and  $C_j$  mutually disjoint.

Let  $P_i \cap P_p \neq \emptyset$ , that means if  $C_i \in 2^{P_i}$  and  $C_p \in 2^{P_p}$  then  $C_i$  and  $C_p$  mutually bicoterie for  $i \neq p, i = 1, 2, \dots, m$  and  $p = 1, 2, \dots, m$ .

Because  $|P_i \cap P_p| = k^2$ , it means that each quorum in  $C_i$  will contain  $k$  elements from  $\{P_i \cap P_p\}$ .

So, from quorum in step 3 we can choose quorum that contains combination  $k$  elements of  $\{P_i \cap P_j\}$ . Finally we can get:

$$\begin{aligned} C_1 &= \{Q_{11}, Q_{12}, \dots, Q_{1k}, \dots\}, & C_1 &\subseteq C_1^* \\ C_2 &= \{Q_{21}, Q_{22}, \dots, Q_{2k}, \dots\}, & C_2 &\subseteq C_2^* \\ &\vdots & & \\ C_m &= \{Q_{m1}, Q_{m2}, \dots, Q_{mk}, \dots\}, & C_m &\subseteq C_m^* \end{aligned}$$

**Example 4.1.**

Let we will construction (2,2)-coterie, then  $n \geq 4|Q|$  and  
 $m \geq h + 2$   
 $m \geq 4$

Next, we find number of  $|Q|$ , let  $m = 4$  then

$$|Q| = \binom{\lfloor \frac{n}{m} \rfloor + 1}{k+1} + (m - h - 1)k$$

$$|Q| = \binom{\lfloor |Q| + 1 \rfloor}{3} + 2 \tag{7}$$

Let  $\frac{\lfloor |Q| + 1 \rfloor}{3} = x$ , then  $|Q| = 3x - 1$ .  
 where

$$x = [x] - \{x\} \tag{8}$$

and

$$0 \leq \{x\} < 1 \tag{9}$$

So, we can write equation (7) as

$$\begin{aligned} 3x - 1 &= [x] + 2 \\ 3([x] - \{x\}) - 1 &= [x] + 2 \\ 3[x] - 3\{x\} - 1 &= [x] + 2 \\ 2[x] - 3 &= 3\{x\} \\ \frac{2}{3}[x] - 1 &= \{x\} \end{aligned} \tag{10}$$

From equation (9) and (10) we get

$$\begin{aligned} 0 &\leq \frac{2}{3}[x] - 1 = \{x\} < 1 \\ 3 &\leq 2[x] < 6 \\ \frac{3}{2} &\leq [x] < 3 \\ [x] &= 2 \end{aligned}$$

Because  $\frac{\lfloor |Q| + 1 \rfloor}{3} = [x] = 2$  then from equation (7) we get

$$|Q| = 4$$

and

$$\begin{aligned} n &\geq 4|Q| \\ n &\geq 16 \end{aligned}$$

So, for arrange (2,2)-coterie then  $n \geq 16$  and  $m \geq 4$ .

Let  $P = \{1, 2, \dots, 16\}$  we can get (4,2,2)-coterie as following

$$\begin{aligned} P_1 &= \{1, 2, 3, 4\} \\ P_2 &= \{5, 6, 7, 8\} \\ P_3 &= \{9, 10, 11, 12\} \\ P_4 &= \{13, 14, 15, 16\} \end{aligned}$$

Since  $h = 2$  then exist  $P_1 \cap P_2 = \emptyset, P_1 \cap P_3 \neq \emptyset$  and  $P_1 \cap P_4 \neq \emptyset$ .

Let we choose  $P_1 \cap P_3 = \emptyset$ , then

$$\begin{aligned} P_1^* &= P_1 \cup P_2 = \{1, 2, 3, 4, 5, 6, 7, 8\} \\ P_3^* &= P_3 \cup P_4 = \{9, 10, 11, 12, 13, 14, 15, 16, \} \end{aligned}$$

In order for satisfy bicoterie property then  $P_1^* \cap P_2^* \neq \emptyset, P_2^* \cap P_3^* \neq \emptyset, P_3^* \cap P_4^* \neq \emptyset$  and  $P_1^* \cap P_4^* \neq \emptyset$ .  
 So that, we get

$$\begin{aligned} P_1^* &= P_1 \cup P_2 = \{1,2,3,4,5,6,7,8\} \\ P_2^* &= P_1 \cup P_3 = \{1,2,3,4,9,10,11,12\} \\ P_3^* &= P_3 \cup P_4 = \{9,10,11,12,13,14,15,16\} \\ P_4^* &= P_2 \cup P_4 = \{5,6,7,8,13,14,15,16\} \end{aligned}$$

Next, we get form of coterie where satisfy  $k$ -coterie property with number of quorum  $|Q| = 4$ .

$$\begin{aligned} C_1^* &= \{\{1,2,3,4\}, \{1,2,3,5\}, \dots, \{4,6,7,8\}, \{5,6,7,8\}\} \\ C_2^* &= \{\{1,2,3,4\}, \{1,2,3,9\}, \dots, \{9,10,11,12\}\} \\ C_3^* &= \{\{9,10,11,12\}, \{9,10,11,13\}, \dots, \{13,14,15,16\}\} \\ C_4^* &= \{\{5,6,7,8\}, \{5,6,7,13\}, \dots, \{13,14,15,16\}\} \end{aligned}$$

From quorum system above, we must choose quorum based combination of elements of  $\{P_i^* \cap P_j^*\}$ , where  $C_i \subseteq P_i^*$  and  $C_j \subseteq P_j^*$  mutually bicoterie.

Let  $C_1 \subseteq P_1^*$  and  $C_2 \subseteq P_2^*$  mutually bicoterie then we take quorum  $C_1$  from  $C_1^*$  which based combination of  $\{1,2,3,4\}$ . Since  $k = 2$ , then each quorum in  $C_1$  contained combination of two number of  $\{1,2,3,4\}$ . That combination means  $\{1,2\}, \{2,3\}, \{3,4\}, \{1,4\}$ .

Therefore we get quorum system as following:

$$\begin{aligned} C_1 &= \{\{1,2,5,6\}, \{3,4,7,8\}, \{1,2,7,8\}, \{3,4,5,6\}, \{1,2,5,8\}, \{3,4,6,7\}, \{1,4,7,8\}, \{2,3,5,6\}, \{1,4,5,8\}, \\ &\{2,3,6,7\}, \{1,4,5,6\}, \{2,3,7,8\}, \{1,2,6,7\}, \{3,4,5,8\}, \{1,4,6,7\}, \{2,3,5,8\}\} \\ C_2 &= \{\{1,3,9,11\}, \{2,4,10,12\}, \{1,4,10,12\}, \{2,3,9,11\}, \{1,3,10,12\}, \{2,4,9,11\}, \{1,4,9,12\}, \\ &\{2,3,10,11\}, \{1,3,10,11\}, \{2,4,9,12\}, \{2,3,10,12\}, \{1,4,9,11\}, \{1,3,9,12\}, \{2,4,10,11\}, \\ &\{1,4,10,11\}, \{2,3,9,12\}\} \\ C_3 &= \{\{9,10,13,14\}, \{11,12,15,16\}, \{9,10,13,16\}, \{11,12,14,15\}, \{9,10,15,16\}, \{11,12,13,14\}, \\ &\{9,12,15,16\}, \{10,11,13,14\}, \{9,12,13,16\}, \{10,11,14,15\}, \{9,10,14,15\}, \{11,12,13,16\}, \\ &\{9,12,13,14\}, \{10,11,15,16\}, \{9,12,15,16\}, \{10,11,13,16\}\} \\ C_4 &= \{\{5,7,13,15\}, \{6,8,14,16\}, \{5,8,14,16\}, \{6,7,13,15\}, \{5,7,14,16\}, \{6,8,13,15\}, \{5,7,13,16\}, \\ &\{6,8,14,15\}, \{5,8,13,15\}, \{6,7,14,15\}, \{5,7,14,15\}, \{6,8,13,16\}, \{5,8,14,15\}, \{6,7,13,16\}, \{5,7,14,16\}, \\ &\{6,8,13,15\}\} \end{aligned}$$

**Theorem 1.** Minimum value of  $n$  for quorum system  $(h, k)$ -Majority coterie are

$$n = \begin{cases} (hk)^2, & m = 2h \\ hk^2(h+1), & m = 2h+1 \end{cases}$$

**Proof.**

For  $m = 2h$ , and  $n = hk|Q|$ .

$$\begin{aligned} |Q| &= \left\lceil \frac{\left\lfloor \frac{n}{m} \right\rfloor + 1}{k+1} \right\rceil + (m-h-1)k \\ &= \left\lceil \frac{\left\lfloor \frac{hk|Q|}{2h} \right\rfloor + 1}{k+1} \right\rceil + (2h-h-1)k \\ &= \left\lceil \frac{k|Q|+2}{2k+2} \right\rceil + (h-1)k \end{aligned}$$

Let  $x = \frac{k|Q|+2}{2k+2}$  then  $|Q| = \frac{(2k+2)x-2}{k}$

So we get

$$\begin{aligned} \frac{(2k+2)x-2}{k} &= [x] + (h-1)k \\ (2k+2)([x] - \{x\}) - 2 &= k[x] + k^2(h-1) \\ \frac{(2k+k)[x] - 2 - k^2(h-1)}{2(k+1)} &= \{x\} \end{aligned}$$

$$0 \leq \frac{(2k+k)[x] - 2 - k^2(h-1)}{2(k+1)} < 1$$

$$\frac{k^2(h-1) + 2}{k+2} \leq [x] < \frac{k^2(h-1) + 2k}{k+2}$$

$$k \leq [x] < \dots$$

Finally we get minimum  $[x] = k$ , then  $|Q| = hk$ .  
 So,  $n = hk|Q| = (hk)(hk) = (hk)^2$ .

For the same way we can proof when  $m = 2h + 1$ .

**Theorem 2.** If  $m$  even ( $m = 2h$ ) then quorum size of  $(h, k)$ -majority coterie equals to quorum size of  $(hk)$ -majority coterie for the same value  $n$ .

**Proof.** From the Theorem 1, when  $m$  is even, we get  $n = (hk)^2$  and  $|Q| = hk$ .  
 So that, for  $(hk)$ -majority coterie we get

$$|Q| = \left\lceil \frac{n+1}{hk+1} \right\rceil = \left\lceil \frac{(hk)^2+1}{hk+1} \right\rceil = hk$$

**Theorem 3.** Coterie and quorum system size

Let  $B = \{C_1, C_2, \dots, C_m\}$  is quorum system  $(h, k)$ -majority coterie then:

$$\|C\| = k^3(|Q| - k), \forall C \in B$$

and

$$\|B\| = m\|C\|$$

**Proof.** From construction quorum step for quorum system  $(h, k)$ -majority coterie we find that if  $C_i \subseteq P_i$  and  $C_j \subseteq P_j$  mutually bicoterie then  $|P_i \cap P_j| = k^2$ .

Each quorum in coterie  $C_i$  contained combination of  $\{P_i \cap P_j\}$  which join with combination of  $\{P_i - (P_i \cap P_j)\}$ . Finally we get:

$$\begin{aligned} \|C\|_i &= |P_i \cap P_j| \cdot (|P_i| - |P_i \cap P_j|) \\ &= k^2 \left( \left\lceil \frac{n}{h} \right\rceil - k^2 \right) \\ &= k^2 \left( \left\lceil \frac{hk|Q|}{h} \right\rceil - k^2 \right) \\ &= k^2(k|Q| - k^2) \\ &= k^3(|Q| - k) \end{aligned}$$

Since each quorum has same size then we get

$$\|C\| = k^3(|Q| - k), \forall C \in B$$

and

$$\|B\| = m\|C\|$$

## 5. Conclusion

In this paper, we get  $|Q| = \left\lceil \frac{n}{m} + 1 \right\rceil + (m - h - 1)k$  to find the size quorum  $(h, k)$ -majority coterie.

Then we find the way for construction quorum system as following: (a) Find the element of universe set  $P_i$ , (b) Arrange  $P_i$  so that we can get coterie which satisfies disjoint and bicoterie property, (c) Construction coterie which satisfy  $k$ -coterie property and quorum size  $|Q|$ , (d) Construction quorum system which satisfy bicoterie and disjoint property. we can make the construction of quorum system  $(h, k)$ -majority if  $m \geq 2h$  and  $n \geq |Q| \cdot hk$ .

## Reference

- [1] Molina, H.G. dan Barbara, D. 1985. How to assign votes in a distributed system. *Journal of The ACM*, **32**(4) 841-860
- [2] Kakugawa, H., Fujita, S., Yamashita, M., dan Ae, T. 1992. A distributed  $k$ -mutual exclusion algorithm using  $k$ -coterie. *Elsevier Information Processing Letters* **49** 213-218
- [3] Ibaraki, T. dan Kameda, T. 1993. A teory of coteries: mutual exclusion in distributed systems. *IEEE Transaction on Parallel and Distributed Computing* (4) 779-794
- [4] Joung, Y. 2004. On quorum systems for group resources with bounded capacity. *LNCS 3274 Distributed Computing* 86-101
- [5] Lawi, A., Oda, K., dan Yoshida, T. 2006. A quorum based  $(m, h, k)$ -resource allocation algorithm. *Conf. on Parallel Dist. Proc. Tech & Appl.* 399-405
- [6] Joung, Y. 2010. On quorum systems for group resources allocation. *Distributed Computing* **22** 197-214
- [7] Maekawa, M. 1985. A  $\sqrt{n}$  algorithm for mutual exclusion in decebtralized systems. *ACM Trans. Comput. Systems* **3**(2) 145-159

# Construction of (h,k)-Coterie Quorum System Based on Majority Coterie

## ORIGINALITY REPORT

16%

SIMILARITY INDEX

12%

INTERNET SOURCES

15%

PUBLICATIONS

10%

STUDENT PAPERS

## PRIMARY SOURCES

1	<a href="http://research.aalto.fi">research.aalto.fi</a> Internet Source	7%
2	Armin Lawi. "Quorum Based Distributed Conflict Resolution Algorithm for Bounded Capacity Resources", Lecture Notes in Computer Science, 2006 Publication	3%
3	<a href="http://digilib.uinsby.ac.id">digilib.uinsby.ac.id</a> Internet Source	1%
4	Hirotsugu Kakugawa. "A Token-Based Distributed Algorithm for the Generalized Resource Allocation Problem", Lecture Notes in Computer Science, 2010 Publication	1%
5	<a href="http://www.navigators.di.fc.ul.pt">www.navigators.di.fc.ul.pt</a> Internet Source	1%
6	Russell D. Blyth, Julianne G. Rainbolt. "Discovering Theorems in Abstract Algebra Using the Software ", PRIMUS, 2010	1%

**7** Yuh-Jzer Joung. "On quorum systems for group resources allocation", Distributed Computing, 02/16/2010 **1%**

Publication

---

**8** Ismail M. Nur, Armin Lawi. "A formal model of quorum based k-mutex algorithm using input/output automata", 2016 International Conference on Computational Intelligence and Cybernetics, 2016 **<1%**

Publication

---

**9** [management.ntu.edu.tw](http://management.ntu.edu.tw) **<1%**

Internet Source

---

**10** [oar.princeton.edu](http://oar.princeton.edu) **<1%**

Internet Source

---

**11** A.W. Fu, W.K. Lau, F.K. Ng, M.H. Wong. "Hypercube quorum consensus for mutual exclusion and replicated data management", Computers & Mathematics with Applications, 1998 **<1%**

Publication

---

**12** [ww1.ucmss.com](http://ww1.ucmss.com) **<1%**

Internet Source

---

Exclude quotes On

Exclude bibliography On

Exclude matches < 5 words